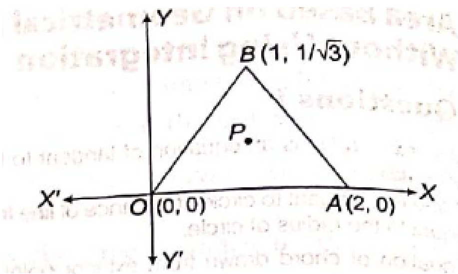
71. $\left(2-\sqrt{3}\right)$ Let the coordinates of P be (x, y)



Equation of line OA be $y = 0$

Equation of line OB be $\sqrt{3}\ y = x$.

Equation of line AB be $\sqrt{3}y = 2 - x$.

$d(P, OA) =$ Distance of P from line OA = y

$d(P, OB) =$ Distance of P from line OB $= \dfrac{\left|\sqrt{3}y - x\right|}{2}$

$d(P, AB) =$ Distance of P from line $AB = \dfrac{\left|\sqrt{3}y + x - 2\right|}{2}$

Given, $d(P, OA) \le \min\{d(P, OB), d(P, AB)\}$

$$y \le \min\left\{\frac{\left|\sqrt{3}y - x\right|}{2}, \frac{\left|\sqrt{3}y + x - 2\right|}{2}\right\} \quad \Rightarrow \quad y \le \frac{\left|\sqrt{3}y - x\right|}{2} \text{ and } y \le \frac{\left|\sqrt{3}y + x - 2\right|}{2}$$

Case – I : When $y \le \dfrac{\left|\sqrt{3}y - x\right|}{2}$ [Since, $\sqrt{3}y - x < 0$]

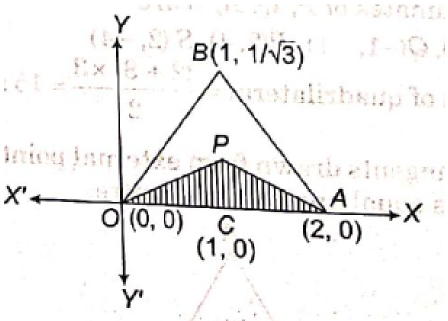
$$y \le \frac{x - \sqrt{3}y}{2} \quad \Rightarrow \quad \left(2+\sqrt{3}\right)y \le x$$

$\Rightarrow \quad y \le x \tan 15°$

Case II: When $y \le \dfrac{\left|\sqrt{3}y + x - 2\right|}{2}$, $2y \le 2 - x - \sqrt{3}y$ [Since, $\sqrt{3}y + x - 2 < 0$]

$\Rightarrow \quad \left(2+\sqrt{3}\right)y \le 2 - x \quad \Rightarrow \quad y \le \tan 15° \,(2 - x)$

From above discussion, P moves inside the triangles as shown below:



$\Rightarrow$ Area of shaded region = Area of $\Delta OPA = \dfrac{1}{2}(Base) \times (Height)$

$= \dfrac{1}{2}(2)\,(\tan 15°) = \tan 15° = \left(2-\sqrt{3}\right)$ square units.

72.(B) Given, $y^3 - 3y + x = 0 \quad \Rightarrow \quad 3y^2\dfrac{dy}{dx} - 3\dfrac{dy}{dx} + 1 = 0 \quad \Rightarrow \quad 3y^2\left(\dfrac{d^2y}{dx^2}\right) + 6y\left(\dfrac{dy}{dx}\right)^2 - 3\dfrac{d^2y}{dx^2} = 0$

At $x = -10\sqrt{2}, y = 2\sqrt{2}$

On substituting in equation (i) we get

$$3\left(2\sqrt{2}\right)^2 \cdot \frac{dy}{dx} - 3 \cdot \frac{dy}{dx} + 1 = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{-1}{21}$$

Again, substituting in equation (ii), we get

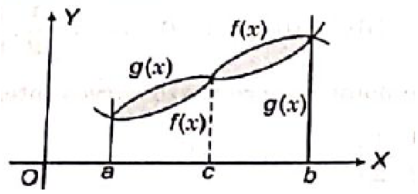
$$3\left(2\sqrt{2}\right)^2 \frac{d^2y}{dx^2} + 6\left(2\sqrt{2}\right)\cdot\left(-\frac{1}{21}\right)^2 - 3\cdot\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \quad 21\cdot\frac{d^2y}{dx^2} = -\frac{12\sqrt{2}}{(21)^2} \quad \Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{12\sqrt{2}}{(21)^3} = \frac{-4\sqrt{2}}{7^3 \cdot 3^2}$$

73.(A) Required area $= \displaystyle\int_a^b y\,dx = \int_a^b f(x)dx = \left[f(x)\cdot x\right]_a^b - \int_a^b f'(x)x\,dx = bf(b) - af(a) + \int_a^b \frac{x\,dx}{3\left[\{f(x)\}^2 - 1\right]}$

$$\left[\because \quad f'(x) = \frac{dy}{dx} = \frac{-1}{3(y^2-1)} = \frac{-1}{3\left[\{f(x)\}^2 - 1\right]}\right]$$

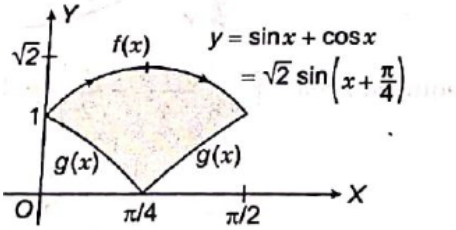
74.(B) To find the bounded area between $y = f(x)$ and $y = g(x)$ between $x = a$ to $x = b$.



$\therefore \quad$ Area bounded $= \displaystyle\int_a^c [g(x) - f(x)]\,dx + \int_c^b [f(x) - g(x)]\,dx = \int_a^b |f(x) - g(x)|\,dx$

Here, $f(x) = y = \sin x + \cos x$, when $0 \le x \le \dfrac{\pi}{2}$ and $g(x) = y = |\cos x - \sin x| = \begin{cases} \cos x - \sin x, & 0 \le x \le \dfrac{\pi}{4} \\ \sin x - \cos x, & \dfrac{\pi}{4} \le x \le \dfrac{\pi}{2} \end{cases}$

could be shown as



$\therefore \quad$ Area bounded $= \displaystyle\int_0^{\pi/4} \{(\sin x + \cos x) - (\cos x - \sin x)\}dx + \int_{\pi/4}^{\pi/2} \{(\sin x + \cos x) - (\sin x - \cos x)\}dx$

$= \displaystyle\int_0^{\pi/4} 2\sin x\,dx + \int_{\pi/4}^{\pi/2} 2\cos x\,dx = -2[\cos x]_0^{\pi/4} + 2[\sin x]_{\pi/4}^{\pi/2}$

$= 4 - 2\sqrt{2} = 2\sqrt{2}\left(\sqrt{2} - 1\right)$ square units.

75.(C) $R_1 = \int_{-1}^{2} x \ f(x)\,dx$ ... (i)

Using $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$R_1 = \int_{-1}^{2} (1-x)\ f(1-x)\,dx$$

$\therefore \quad R_1 = \int_{-1}^{2} (1-x)\ f(x)\,dx$ ... (ii)

$[f(x) = f(1-x), \text{given}]$

Given, R_2 is area bounded by $f(x), x = -1$ and $x = 2$.

$\therefore \quad R_2 = \int_{-1}^{2} f(x)\,dx$ ... (iii)

On adding equation (i) and (ii), we get $2R_1 = \int_{-1}^{2} f(x)\,dx$ ... (iv)

From equation (iii) and (iv), we get $2R_1 = R_2$

76.(B) Here, area between 0 to b is R_1 and b to 1 is R_2.

$\therefore \quad \int_0^b (1-x)^2\,dx - \int_b^1 (1-x)^2\,dx = \frac{1}{4}$

$\Rightarrow \quad \left[\frac{(1-x)^3}{-3}\right]_0^b - \left[\frac{(1-x)}{-3}\right]_b^1 = \frac{1}{4} \quad \Rightarrow \quad -\frac{1}{3}[(1-b)^3 - 1] + \frac{1}{3}[0-(1-b)^3] = \frac{1}{4}$

$\Rightarrow \quad -\frac{2}{3}(1-b)^3 = -\frac{1}{3} + \frac{1}{4} = -\frac{1}{12} \quad \Rightarrow \quad (1-b)^3 = \frac{1}{8} \quad \Rightarrow \quad (1-b) = \frac{1}{2} \quad \Rightarrow \quad b = \frac{1}{2}$

77.(B) Required area $= \int_0^{\pi/4} \left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\right) dx \qquad \left[\because \quad \frac{1+\sin x}{\cos x} > \frac{1-\sin x}{\cos x} > 0\right]$

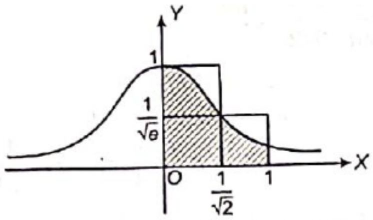
$$= \int_0^{\pi/4} \left(\sqrt{\frac{1+\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\dfrac{1-\tan x^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}} - \sqrt{\frac{1-\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}} \right) dx$$

$$= \int_0^{\pi/4} \left(\sqrt{\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}} - \sqrt{\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}} \right) dx = \int_0^{\pi/4} \frac{1+\tan\frac{x}{2} - 1 + \tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}\,dx = \int_0^{\pi/4} \frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}\,dx$$

Put $\tan\frac{x}{2} = t \quad \Rightarrow \quad \frac{1}{2}\sec^2\frac{x}{2}\,dx = dt = \int_0^{\tan\frac{\pi}{8}} \frac{4t\,dt}{(1+t^2)\sqrt{1-t^2}}$

As $\int_0^{\sqrt{2}-1} \frac{4t\,dt}{(1+t^2)\sqrt{1-t^2}} \qquad [\because \quad \tan\frac{\pi}{8} = \sqrt{2}-1]$

78.(BD) Graph for $y = e^{-x^2}$



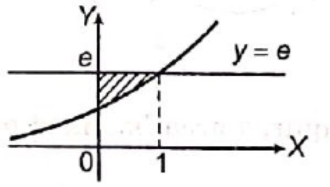
Since, $x^2 \le x$ when $x \in [0, 1]$

$\Rightarrow \quad -x^2 \ge -x \quad$ or $\quad e^{-x^2} \ge e^{-x}$

$\therefore \quad S \ge -(e^{-x})_0^1 = 1 - \frac{1}{e} \quad$ ... (i)

Also, $\int_0^1 e^{-x^2}\, dx \le$ Area of two rectangles $\le \left(1 \times \frac{1}{\sqrt{2}}\right) + \left(1 - \frac{1}{\sqrt{2}}\right) \times \frac{1}{\sqrt{e}} \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$ ... (ii)

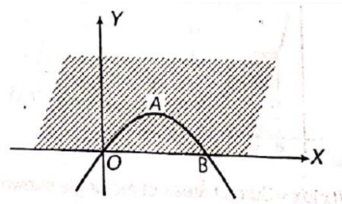
79.(BCD) Shaded area $= e - \left(\int_0^1 e^x dx\right) = 1$



Also, $\int_1^e \ln\,(e + 1 - y)dy \quad$ [Put $e + 1 - y = t \quad \Rightarrow \quad -dy = dt$]

$= \int_e^1 \ln\, t(-dt) = \int_1^e \ln\, t\ dt = \int_1^e \ln\, y\ dy$

80.(BD) Case I: When $m = 0$



In this case, $y = x - x^2 \quad$ ... (i)

and $y = 0 \quad$ ... (ii)

are two given curves, $y > 0$ is total region above X-axis.

Therefore, area between $y = x - x^2$ and $y = 0$

Is area between $y = x - x^2$ and above the X-axis.

$A = \int_0^1 x - x^2 dx = \frac{1}{6}$

Hence no solutions.

Case-II; When $m < 0$ in this case area required is $\int_0^{1-m} x - x^2 - mx\, dx$

$\Rightarrow \quad \frac{9}{2} = \frac{1}{6}(1 - m)^3 \quad$ [Given]

$\Rightarrow \quad (1 - m)^3 = 27 \quad \Rightarrow \quad (1 - m) = 3$

$\Rightarrow \quad m = -2$

Case-III: When $m > 0$ in this case area required is $\int_{1-m}^0 x - x^2 - mx\, dx$

$\Rightarrow \quad \frac{9}{2} = -\frac{1}{6}(1 - m)^3 \quad$ [Given]

$\Rightarrow \quad (1 - m)^3 = -27 \quad \Rightarrow \quad (1 - m) = -3$

$\Rightarrow \quad m = 4$

Therefore, (b) and (d) are the answers.